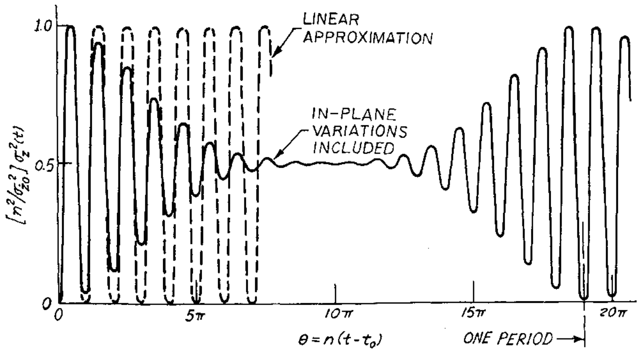

Fig. 1 The variance of out-of-plane position.

Equations (5a) and (6a) are compared graphically in Fig. 1, where the variance $\sigma_z^2(t)$ is plotted as a function of $\theta = n(t - t_0)$. We can readily observe a significant change of the time behavior of the variance due to small dispersions in in-plane parameters. Instead of regular oscillatory property as predicted by linear approximation, the behavior of the variance $\sigma_z^2(t)$ is that analogous to the "beat" phenomenon commonly observed in the vibration of engineering systems. The beat period is approximately $19\pi/n$ for the statistical model assumed for N.

It is clear from Eqs. (6b) and (6c) that the observations just discussed described equally well the behaviors of $\sigma_{\dot{z}}^2(t)$ and $\mu_{z\dot{z}}(t)$.

The restricted problem considered in this note serves to demonstrate that the statistical behavior of the out-of-plane motion of a satellite can deviate significantly from that predicted by linear perturbation theory. For satellites of long lifetimes in particular, errors in in-plane parameters may become important and cannot be ignored in predicting the out-of-plane perturbations.

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Effect of Temperature on Pressure Measurements

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Nomenclature

Kn = Knudsen number = λ/r
p = pressure
r = inside radius of the tube
T = absolute temperature
λ = mean free path of the gas

Subscripts

c = cold end of the tube
h = hot end of the tube
l = large tube
s = small tube

Knudsen[1] has shown that, when two containers at unequal temperatures are connected by a tube, the pressures in the two containers do not become equal if the mean free path of the gas λ is such that $\lambda/r > \frac{1}{100}$ (approximately), where r is the radius of the tube. He defined the equilibrium condition in the tube as being one of zero net mass transfer across a section rather than an equality of pressure. The condition of zero net mass transfer can exist when a flow from the cold to hot end along the wall of the tube is exactly balanced by a flow from hot to cold along the axis of the tube. For small values of mean free path the central flow predominates, and the condition of pressure equality may be attained. When the mean free path is large, the flow close to the wall exercises a significant effect. In the limiting case where the mean free path is very much larger than the tube diameter, the pressures may be related by the approximate, free-molecule flow equation:

$$p_c/p_h = (T_c/T_h)^{1/2} \tag{1}$$

Knudsen conducted experiments to determine the pressure and temperature relationship in the continuum and transition regimes. As a result of these tests, he derived the following relationship:

$$dp/dT = (p/2T)\{1 + 2.46(Kn + 3.15)/[Kn(Kn + 24.6)]\}^{-2} \tag{2}$$

This equation was derived from experiments with hydrogen in glass tubes where the temperature difference between the ends of the tube was small. Howard[2] extended this work to stainless-steel tubing containing air with a much larger temperature difference between the two ends of the tube. For Knudsen numbers greater than two, the agreement between Howard's data and Knudsen's semiempirical equation is not good.

Because of the uncertainty as to the form of the pressure-temperature relationship for Knudsen numbers greater than two, an experimental program designed to better determine this variation has been carried out by the authors. A complete description of the experimental apparatus and procedure is contained in two test reports.[3, 4] Basically, the apparatus consists of two tubes, one large and the other small, joined together as shown in Fig. 1. The temperature of this junction can be controlled by an electrical resistance heater. The other ends of the two tubes were water cooled. Pressures at the cold ends of the tubes were measured with a well-calibrated, low-range, pressure transducer. The temperatures at the hot and cold ends of the tubes were measured with chromel-alumel and copper-constantan thermocouples, respectively.

Inspection of Knudsen's semiempirical equation, Eq. (2), indicates that, for a fixed temperature ratio T_h/T_c, the pressure ratio p_c/p_h is a function of Knudsen number only. This indicates a method of analyzing the test data. Con-

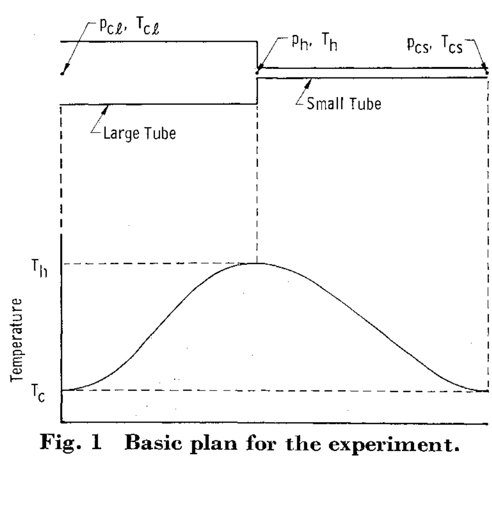

Fig. 1 Basic plan for the experiment.

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sider the measured pressures and temperatures. These values enable the quantities T_h/T_c, Kn_{cl}, and Kn_{cs} to be calculated. From Knudsen's results, it seems reasonable to assume that $p_c = p_h$ when $Kn_c \leqslant 0.01$, or $p_{cl} = p_{hs} = p_h$. Therefore, at each temperature ratio T_h/T_c, the ratio of p_{cs}/p_h may be calculated for a range of values of Kn_{cs} up to the point where $Kn_{cl} = 0.01$. When $Kn_{cl} \geqslant 0.01$, then p_{cl}/p_h is given by the experimentally determined variation of p_{cs}/p_h with Kn_{cs} which was determined when Kn_{cl} was still less than 0.01. Therefore, when p_{cl} is known, p_h and p_{cs}/p_h can be calculated. Thus, by using this bootstrap technique, the relationship between p_{cs}/p_h may be extended to large values of Kn_c.

An example of the results of the experiments is shown in Fig. 2. It can be seen that, up to a Knudsen number of five, the degree of scatter in the data is very small. Since

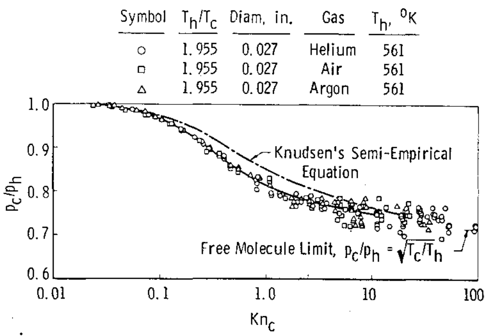

Fig. 2 Variation of cold-to-hot pressure ratio with Knudsen number (cold).

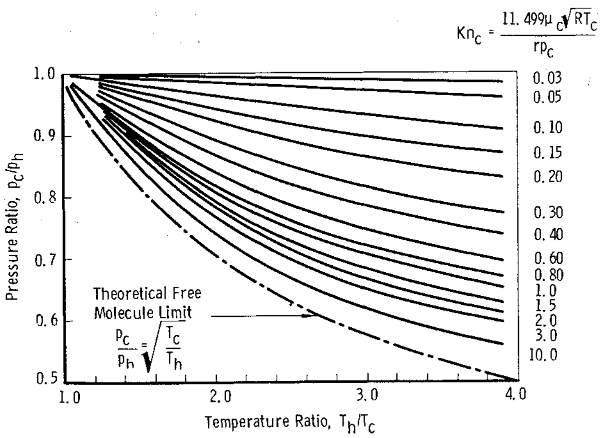

Fig. 3 A working chart to correct for thermomolecular pressure effects in tubes (gage at the cold end).

this test involved the use of three different gases (air, argon, and helium), the results tend to confirm the validity of using Knudsen number as a correlating parameter. For Knudsen numbers greater than five, the degree of scatter increases to approximately ±5% (±0.5 μ Hg in these tests) about a mean curve through the experimental data. Despite this degree of scatter, it seems reasonable to conclude that the limiting value of p_c/p_h predicted by Eq. (1) is attained at a sufficiently large Knudsen number. Furthermore, it will be noted that there is some disagreement between the values predicted by Knudsen's semiempirical equation and the present results. Knudsen's curve was derived on the basis of data for a temperature ratio of 1.07. He did, in fact, carry out some tests at a temperature ratio of approximately two and found that his semiempirical equation did not predict exactly the measured values.

The temperature ratio covered in the author's experiments ranges from 1.5 to 3.8. Using these results, a working chart permitting the calculation of the temperature effect on pressure is given in Fig. 3.

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Viscous Interaction Effects on a Static Pressure Probe at $M = 6$

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The purpose of this investigation was to find the relation between measured static pressure and true static pressure as a function of the viscous interaction parameter $\bar{\chi}$. The results obtained make possible accurate interpretation of measured static pressure data.

1. Design of Pitot-Static Probe

The dimensions of the static pressure probe were chosen so that the measured value of the static pressure was as close as possible to the freestream static pressure. Using the data of Matthews,[1] three pressure orifices were located 10 diameters behind the shoulder of a tube with a 20° cone tip. The length of the uniform-diameter tube section behind the pressure orifices was chosen to be 20 boundary-layer thicknesses (calculated at the location of the orifices, for the lowest operating pressure). A sketch of the probe is shown in Fig. 1.

2. Measurements and Data Reduction

The pitot pressures were measured with a Welch mercury manometer and the static pressures with a silicone U-tube manometer. The total pressures were measured by means of a Tate-Emery pressure indicator. The static pressures of the freestream were computed from the pitot and total pressures using the isentropic flow relations. The accuracy of the pitot pressure measurements was estimated to ±0.2%, and that of the total pressure was ±0.1%, so that the "correct" static pressures were accurate to about ±0.3%. The accuracy of the pressure measured with the static probe was also about ±0.3%. During all tests, the total temperature was maintained at 300°F. (At a total temperature of 234°F and for pressures greater than 50 psig, inconsistent results were obtained, which were traced to condensation effects.) Measurements were taken at total pressures ranging from 8 to 100 psig. The experimental data obtained are plotted in Fig. 2 in terms of the ratio (p_m/p_∞), where p_m is the measured pressure and p_∞ is the "correct" static pressure.

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